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Memory-based event-triggered leader-following consensus for T-S fuzzy multi-agent systems subject to deception attacks

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Abstract

In this paper, the problem of fuzzy model-based leader-following consensus control for multi-agent systems (MASs) under deception attacks is investigated. For the sake of alleviating the communication burden, a novel memory-based event-triggered scheme (METS) is first proposed for the considered MASs to reduce redundant data transmission, and the leader-following consensus can be achieved faster with a smaller adjustment error by applying the historical released packets. Considering the designed METS and upper-bounded attacks synthetically, the closed-loop fuzzy system model is well established. Furthermore, with the help of Lyapunov-Krasovskii technique, some sufficient conditions are derived to ensure the consensus of MASs subject to deception attacks. Finally, a simulation example is introduced to manifest the effectiveness of the proposed method.

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1. Introduction

In the past a few years, the research results of consensus control for multi-agent systems (MASs) are widely used in many fields, such as UAVs formation [1], containment control

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[2,3], tracking control [4,5], flocking [6], opinion dynamics [7] and so forth. By designing the appropriate distributed controllers, the goal of the consensus problem is to make the state of agents becomes normalized in the promise of agents use only the local information exchange [8,9]. Generally, in terms of the existence of leader, consensus control problem discussed in the literature can be classified into leaderless consensus control and leader-following consensus control. With regard to leader-following MASs, the leader has influence on the followers' behaviors, while it is independent of the followers [10,11]. Specifically, the sampled-data leader-following consensus control was investigated for a class of nonlinear MASs in [12]. In consideration of unknown periodic time-varying parameters and inaccurate communication topology, the consensus control problem of the second-order nonlinear MASs was addressed in [13]. The authors in [14] investigated the adaptive leader-following consensus control for uncertain nonlinear MASs with single- and double-integrator leader, respectively.

Recently, the Takagi-Sugeno (T-S) fuzzy model has been extensively used to settle the consensus control problem that the connection between nodes is usually indeterminate due to the complexity and uncertainty of the MASs. It is widely accepted that the T-S fuzzy model is an efficient solution to nonlinear control problems, as it is recognized as an effective way to approximate uncertain nonlinear mapping with unstructured uncertainty [15]. Taking an imprecise communication topology structure into account, the consensus control problem for a high-order MASs described by fuzzy model was investigated in [16]. As pointed in [17], the T-S fuzzy model has been used to describe the MASs considering stochastic disturbance and time-varying delay.

To our knowledge, the event-triggered scheme (ETS) has attracted much focus due to the promising advantage of less resource utilization [18–22]. For a MAS, event-triggered controller is applied to each follower for the sake of decreasing dispensable communication, which decides whether the sampled data of follower should be broadcast to the others [23]. The event-triggered consensus problem of MASs without and with time delay was studied in [24], where the sampling period is permitted to be arbitrarily large by using the designed algorithm. Applying a distributed ETS, the problem of leader-following consensus control was investigated in [25], in which agents exchange information via a limited communication medium. With regard to the issue of finite-time consensus, the authors in [27] adopted a distributed ETS with model-based triggering function to ensure the finite-time convergence of the disagreement vector and the triggering error. In [26], the bipartite consensus problem of MAS was studied, and intermittent communication could be implemented by utilizing observer-based control scheme. Based on the ETS together with the networked predictive control scheme, the authors in [28] proposed an event-triggered predictive control protocol for distributed adaptive system to compensate for the communication delays actively. Nevertheless, it should be mentioned that the application of historical-information-based ETS for the consensus issue of MASs is seldom discussed. As described in the literatures [29,30], the demand of more packets near the trough or vertex of the response curve to shorten the response fluctuation can be met with the help of historic released packets. Meanwhile, several recently transmitted sampled data packets are conducive to decide the following possible triggering event. The authors in [31] proposed an ETS based on the history of the measured outputs, in which a simple link between the parameters of the sampling criterion and the speed of convergence was given. Additionally, by the use of enforcing a minimum inter-event time, accumulation of sampling times was prevented. Hence, this gives impetus to present investigation.

Due to the increased reliance on communication network, the MASs are vulnerable to malicious attack injection, and such problem may deteriorate the consensus control performance [32–34]. As a result, many scholars are devoted to the research of cyber attacks, and a large number of achievements on cyber attacks have been available. For example, to deal with MASs with lossy sensors and cyber attacks, the consensus control problem was investigated in [33]. In [35], the random deny-of-service attack was discussed in the consensus problem under a distributed control law. With regard to a class of nonlinear MASs hit by deception attacks, a sampled-based consensus control problem was discussed in [36]. In the framework of impulsive control, the secure synchronization of MASs subject to deception attacks was studied in [37]. In [38], by taking different attack intensity into account, consensus control problem was studied for heterogeneous MASs suffering denial-of-service attack. Significantly, more data packets need to be transmitted to obtain the better consensus control performance when MASs subject to attacks, which motivates the current study.

Inspired by the mentioned works, in this study, we mainly focus on the problem of fuzzy model-based leader-following consensus control for MASs subject to deception attacks. However, the existed METS is not applicable for MASs. Then, how to integrate the METS into the research of consensus control problem to ensure control performance and save bandwidth brings new opportunities and challenges. In order to address this problem, we combine METS with MASs to study the issue of consensus control, and some recent released data are utilized in the proposed scheme to increase the transmission instants at the crest or trough of the responses [29]. In comparison with the existing works, The major contributions of this paper are highlighted as follows:

- (1) A novel METS applying the historical data packets of followers is designed for consensus control, under which the MASs can be more sensitive to deception attacks. Different from memoryless controller design method in [39], when malicious attacks are launched on the agents, the data-releasing rate increases such that controller receives more information to improve the control performance.
- (2) Based on the proposed METS, a fuzzy model-based piecewise control strategy is proposed for leader-following consensus of MAS such that a better performance can be further achieved.
- (3) Sufficient conditions for the stability of the T-S fuzzy MASs considering both the METS and the impact of deception attacks are derived, and the piecewise fuzzy controller are obtained by solving a series of linear matrix inequalities.

The rest of this paper is structured as follows. The problem formulation including T–S fuzzy based multi-agent model, deception attacks and METS is described in Section 2. In Section 3, some sufficient conditions are established to guarantee the desired control performance and the leader-following consensus in secure of the MASs with the METS and deception attacks, and the design method of controllers is given. In addition, a simulation example is introduced to manifest the proposed method in Section 4. Finally, the conclusion is drawn in Section 5.

Notation: $\text{diag}_N\{U\}$ (or $\text{diag}_N\{U_i\}$) denotes the N -block-diagonal matrix $\text{diag}\{U, \dots, U\}$ (or $\text{diag}\{U_1, \dots, U_N\}$). Similarly, $\text{col}_N\{U\}$ (or $\text{col}_N\{U_i\}$) denotes the column vector $\text{col}\{U, \dots, U\}$ (or $\text{col}\{U_1, \dots, U_N\}$). \otimes indicates the Kronecker product for matrices. \mathbb{R}^n and $\mathbb{R}^{N \times N}$ denote the n -Euclidean space and the set of all $N \times N$ dimensional real matrices. Besides, I_n and I_N represent the identity matrix of n order and N order, respectively. $*$ stands for the symmetric elements in symmetric block matrices. $\|\bullet\|$ describes Euclidean norm for vectors or matrices.

2. Problem formulation

The MAS considered in this study consists of one leader and N followers. A directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{C})$ without self-loops is used to describe the communication topology, where $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$ is the index set of N followers; $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denotes the set of edges and $\mathcal{C} = [c_{ij}] \in \mathbb{R}^{N \times N}$ is defined to represent a weighted adjacency matrix where $c_{ij} > 0$ if $(j, i) \in \mathcal{E}$, in this case, the agent i can receive the information of agent j ; otherwise, $c_{ij} = 0$. $\mathcal{N}_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}\}$ is used to express the set of neighbors of the i th follower.

2.1. T–S fuzzy based multi-agent model

The considered i th follower is depicted by the following T–S fuzzy model with r plant rules.

Plant rule m : IF $z_1(t)$ is W_{m1} , and $z_2(t)$ is W_{m2} , and \dots , and $z_q(t)$ is W_{mq} , THEN

$$\begin{cases} \dot{x}_i(t) = A_m x_i(t) + B_m u_i(t) \\ \dot{x}_0(t) = A_m x_0(t) \end{cases}, \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ and $x_0(t) \in \mathbb{R}^n$ denote the state of the i th agent and the leader agent, respectively; $u_i(t) \in \mathbb{R}^m$ represents the control input of the i th agent ($i = 1, 2, \dots, N$); W_{mg} is a fuzzy set ($m \in \{1, 2, \dots, r\}$, $g \in \{1, 2, \dots, q\}$); $z_1(t), z_2(t), \dots, z_q(t)$ are the premise variables. A_m, B_m are constant matrices with compatible dimensions.

The overall model of the MAS is described by

$$\begin{cases} \dot{x}_i(t) = \sum_{m=1}^r h_m(z(t))(A_m x_i(t) + B_m u_i(t)) \\ \dot{x}_0(t) = \sum_{m=1}^r h_m(z(t))A_m x_0(t) \end{cases}, \tag{2}$$

where $h_m(z(t)) = \frac{\mu_m(z(t))}{\sum_{m=1}^r \mu_m(z(t))}$ satisfies $h_m(z(t)) \geq 0$, and $\sum_{m=1}^r h_m(z(t)) = 1$, $\mu_m(z(t)) = \prod_{l=1}^q W_{ml}(z_l(t))$ represents the membership function.

2.2. Deception attacks

Assumption 2.1. A directed graph \mathcal{G} is assumed to contain a directed spanning tree, that is, the leader has direct path to every follower.

Assumption 2.2. The pair (A_m, B_m) is stabilizable.

The information interaction between the agents in this study is transmitted over the communication network (see Fig. 1), which is vulnerable to deception attack. The malicious attacks for transmitted information are designed by

$$\hat{x}_j(t) = x_j(t) + q_j(t), \tag{3}$$

where the attack function $q_j(t) \in \mathbb{R}^n$ is assumed to satisfy

$$\|q_j(t)\|_2 \leq \|G_j(x_j(t) - x_0(t))\|_2, \tag{4}$$

with a known energy constraint matrix G_j .

Remark 2.1. From Eq. (4), one can see that the attacker launches a deception attack by using the information of i th follower $x_i(t)$ and that of the leader $x_0(t)$, which implies that

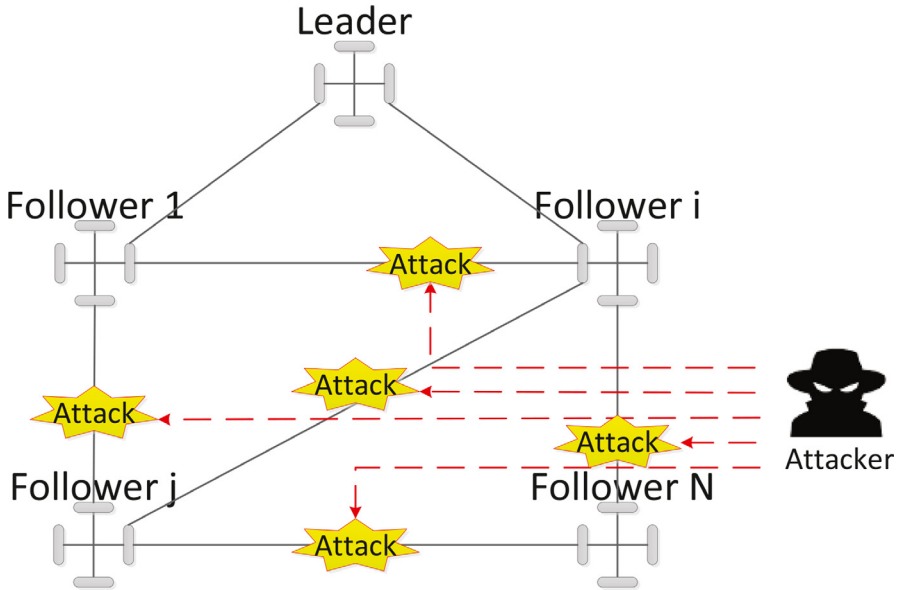


Fig. 1. Deception attacks among agents.

each follower has certain direct relationship with the leader. However, it is hard to meet this strong requirement in the practical application of multi-agent systems. Therefore, the further consideration for weakening this requirement of communication topology for MASs subject to deception attacks will be discussed in the further research.

Remark 2.2. Deception attacks mean to invade communication channels, sending erroneous data packets to the controller which desynchronize followers and the leader. Different from the existing deception attack models, the established attack formed in Eq. (4), where the attack signal using the consensus error between follower and the leader, is closer to the actual situation.

2.3. Memory-based event-triggered scheme

To reduce the number of triggering event of data releasing, A METS is utilized for each agent transmission. Under the METS, the i th agent only broadcasts the data packets to its neighbor when the triggering event is generated.

Inspired by Tian and Peng [29], we take the historical information into account to establish the METS. Under such a METS, the next releasing instant of the i th agent can be expressed by

$$t_{k+1}^i h = t_k^i h + \max\{(l + 1)h | \zeta_i(t) < 0\}, \tag{5}$$

where

$$\zeta_i(t) = \sum_{s=1}^{\hat{s}} \lambda_s e_i^{sT}(t) \Phi_i e_i^s(t) - \delta_{i\hat{\sigma}}^T(t) \Phi_{i\hat{\sigma}}(t),$$

$$e_i^s(t) = x_i(t_{k-s+1}^i h) - \bar{x}_i(t_{k-s+1}^i h),$$

$$\bar{x}_i(t_{k-s+1}^i h) = \frac{x_i(t_{k-s+1}^i h) + x_i(t_k^i h + lh)}{2},$$

$$\mathcal{G}(t) = \frac{1}{\hat{s}} \sum_{s=1}^{\hat{s}} \eta_i(t_{k-s+1}^i h),$$

$$\eta_i(t_{k-s+1}^i h) = \sum_{j \in N_i} c_{ij} (x_i(t_{k-s+1}^i h) - \hat{x}_j(t_{k'-s+1}^j h)) + d_i (x_i(t_{k-s+1}^i h) - x_0(t_{k-s+1}^i h)),$$

and δ_i, λ_s are given positive scalars. $d_i = 1$ denotes that the information of the leader can be available to agent i ; otherwise, $d_i = 0$. \hat{s} and $t_{k-s+1}^i h$ represent the number of historical packets and historic released instant for agent i , respectively. $\Phi_i > 0$ ($i = 1, 2, 3, 4$) is a weight matrix to be designed. $t_k^i h$ is on behalf of the data releasing instant of i th follower, and immediately sampling packets $t_k^i h + lh$ are discarded with $l = 1, 2, \dots$. For brevity, we let $\vartheta h = t_k^i h + lh$. Additionally, $[t_k^i h, t_{k+1}^i h) = \cup_{\vartheta h=t_k^i h}^{(t_{k+1}^i-1)h} [\vartheta h, (\vartheta + 1)h)$.

Remark 2.3. From Eq. (5), it can be clearly observed that an average $\bar{x}_i(t_{k-s+1}^i h)$ of the historic released signal $x_i(t_{k-s+1}^i h)$ and the immediately sampling signal $x_i(t_k^i h + lh)$ is used to develop the METS. It leads to the following advantages: 1) the unnecessary releasing data caused by the abrupt jitter variation of system state are reduced to a great extent; 2) the releasing period is smoothed.

Remark 2.4. In Eq. (5), \hat{s} released packets are used to develop the METS. In the following study, we take the past 3 packets from the buffer to represent the historical information for simplification. In fact, the average method utilized in designing the triggering condition can be extended to the average of i th agent state at multiple instants. However, considering more historical information means that more storage space will be occupied and more computing resources will be consumed. Thus, we select $\hat{s} = 3$ here. If one take \hat{s} as 1, as the exceptional case, the METS degrades into memoryless ETS and fewer information is used to achieve the consensus.

Remark 2.5. It needs to be pointed out that λ_1 in Eq. (5) is set larger than the others in that the new released packet is more important than the old ones. $\mathcal{G}(t)$ takes the mean value of three historical information from the system so that the METS is much more sensitive to deception attacks, thereby reducing the influence of instantaneous releasing signals.

Through the above discussion, for $t \in [\vartheta h, (\vartheta + 1)h)$, we define the consensus error as $\chi_i(\vartheta h) = x_i(\vartheta h) - x_0(\vartheta h), i = 1, 2, \dots, N$. Considering the proposed METS, the fuzzy-model based piecewise controller for p th rule can be established as follows:

Controller rule p : IF $z_1(t)$ is W_{p1}^{ks} , and $z_2(t)$ is W_{p2}^{ks} , and \dots , and $z_q(t)$ is W_{pq}^{ks} , THEN

$$u_i(t) = K_{ps} \eta_i(t_{k-s+1}^i h), \tag{6}$$

where $K_{ps} (p \in \{1, 2, \dots, r\}; s \in \{1, 2, 3\})$ are desired control gains to be determined.

Then the memory-based fuzzy consensus controller is proposed as

$$u_i(t) = - \sum_{p=1}^r \sum_{s=1}^3 h_p^{ks} (z(t_{k-s+1}^i h)) K_{ps} \eta_i(t_{k-s+1}^i h)$$

$$\begin{aligned}
 &= - \sum_{p=1}^r \sum_{s=1}^3 h_p^{ks}(z(t_{k-s+1}^i h)) K_{ps} \left\{ \sum_{j \in N_i} l_{ij} [2e_i^s(t) + \chi_i(\vartheta h)] \right. \\
 &\quad \left. - \sum_{j \in N_i} c_{ij} q_j(t_{k-s+1}^j h) + d_i [2e_i^s(t) + \chi_i(\vartheta h)] \right\}, \tag{7}
 \end{aligned}$$

where membership function $h_p^{ks}(z(t_{k-s+1}^i h)) = \frac{\mu_p^{ks}(z(t_{k-s+1}^i h))}{\sum_{p=1}^r \mu_p^{ks}(z(t_{k-s+1}^i h))}$ satisfies $h_p^{ks}(z(t_{k-s+1}^i h)) \geq 0$, $l_{ij} = -c_{ij}$, and the membership functions are subject to

$$h_p^{ks}(z(t_{k-s+1}^i h)) \geq \rho_p h_p(z(t)), \tag{8}$$

where $\rho_p > 0$, ($p = 1, 2, \dots, r$). In the following, for brevity, we define $h_m(z(t)) \triangleq h_m$, $h_p^{ks}(z(t_{k-s+1}^i h)) \triangleq h_p^{ks}$.

Remark 2.6. Equation (8) is used to deal with the issue of imperfect premise matching between the T-S fuzzy MAS and the fuzzy-based consensus controller.

Remark 2.7. Considering the proposed METS with \hat{s} historical informations, each historical information introduced to the controller is matched with different controller gain to get a satisfied control performance as described in Eq. (7).

2.4. The overall model

From the definition of $e_i^s(t)$, the closed-loop error system can be formulated as

$$\begin{aligned}
 \dot{\chi}_i(t) &= \sum_{m=1}^r \sum_{p=1}^r \sum_{s=1}^3 h_m h_p^{ks} [A_m \chi_i(t) - B_m K_{ps} \sum_{j \in N_i} l_{ij} (2e_i^s(t) + \chi_i(\vartheta h))] \\
 &\quad + B_m K_{ps} \sum_{j \in N_i} c_{ij} q_j(t_{k-s+1}^j h) - d_i B_m K_{ps} (2e_i^s(t) + \chi_i(\vartheta h)), \tag{9}
 \end{aligned}$$

for $t \in [\vartheta h, (\vartheta + 1)h)$, where

$$\begin{aligned}
 \tilde{A} &= \sum_{m=1}^r h_m A_m, \tilde{B} = \sum_{m=1}^r h_m B_m, \hat{K}_s = \sum_{s=1}^3 \sum_{p=1}^r h_p^{ks} K_{ps}, H = L + D, \\
 C &= \begin{bmatrix} 0 & * & \dots & * \\ c_{21} & 0 & \dots & * \\ \vdots & \vdots & \ddots & \vdots \\ c_{N1} & c_{N2} & \dots & 0 \end{bmatrix}, D = \text{diag}_N \{d_i\}, L = (l_{ij})_{N \times N} \text{ with } l_{ii} = \sum_{j \in N_i} c_{ij}, l_{ij} = -c_{ij}.
 \end{aligned}$$

Define $\tau_i(t) = t - \vartheta h$ for $t \in [\vartheta h, (\vartheta + 1)h)$, $\tau_i(t)$ satisfies $\tau_i(t) \in [0, h)$ with the derivative $\dot{\tau}_i(t) = 1$ at $t \neq \vartheta h$.

By using Kronecker product, the overall error dynamic can be rewritten as

$$\dot{\chi}(t) = (I_N \otimes \tilde{A})\chi(t) - (H \otimes \tilde{B}\hat{K}_s)F(t) - 2(H \otimes \tilde{B}\hat{K}_s)e^s(t) + (C \otimes \tilde{B}\hat{K}_s)q(t_{k-s+1}h), \tag{10}$$

for $t \in [\vartheta h, (\vartheta + 1)h)$, where

$$\chi(t) = \text{col}\{\chi_1(t), \chi_2(t), \dots, \chi_N(t)\},$$

$$\begin{aligned}
 F(t) &= \text{col}\{\chi_1(t - \tau_1(t)), \chi_2(t - \tau_2(t)), \dots, \chi_N(t - \tau_N(t))\}, \\
 e^s(t) &= \text{col}\{e_1^s(t), e_2^s(t), \dots, e_N^s(t)\}, \\
 q(t_{k-s+1}h) &= \text{col}\{q_1(t_{k-s+1}^1h), q_2(t_{k-s+1}^2h), \dots, q_N(t_{k-s+1}^Nh)\}.
 \end{aligned}$$

For simplicity, the system Eq. (10) can be rewritten as the following augmented form:

$$\dot{\chi}(t) = \mathcal{A}\zeta(t), \tag{11}$$

for $t \in [\vartheta h, (\vartheta + 1)h)$, where

$$\begin{aligned}
 \mathcal{A} &= [I_N \otimes \tilde{A} \quad -H \otimes \tilde{B}\hat{K}_s \quad 0 \quad \mathcal{A}_e \quad \mathcal{A}_q - 2H \otimes \tilde{B}\hat{K}_1 \quad C \otimes \tilde{B}\hat{K}_1], \\
 \zeta(t) &= \text{col}\{\chi(t), F(t), \chi(t - h), f_e(t), f_q(t)\},
 \end{aligned}$$

with $\mathcal{A}_e = [-2H \otimes \tilde{B}\hat{K}_1 \quad -2H \otimes \tilde{B}\hat{K}_2 \quad -2H \otimes \tilde{B}\hat{K}_3]$, $\mathcal{A}_q = [C \otimes \tilde{B}\hat{K}_1 \quad C \otimes \tilde{B}\hat{K}_2 \quad C \otimes \tilde{B}\hat{K}_3]$, $f_e(t) = \text{col}\{e^1(t), e^2(t), e^3(t)\}$, $f_q(t) = \text{col}\{q(t_kh), q(t_{k-1}h), q(t_{k-2}h)\}$.

The objective of this paper is to design a fuzzy-model based consensus controller for the MAS applying the memory-based ETS, such that the MAS subject to the deception attacks may maintain the performance at a certain level.

3. Main results

Before further derivation, the following definition and lemma are needed.

Definition 3.1. [40] The leader-following consensus of MAS Eq. (2) with the designed controller Eq. (7) is achieved, if the following criterion

$$\lim_{t \rightarrow \infty} \|x_i(t) - x_0(t)\| = 0, i = 1, 2, \dots, N, \tag{12}$$

holds for any initial conditions $x_i(0)$.

Lemma 3.1. [41] For a given constant $h > 0$, if there exist constant matrix $R = R^T > 0 \in \mathbb{R}^n$, satisfying $F = \text{diag}\{I_N \otimes R, 3(I_N \otimes R)\} > 0$, then we have

$$-h \int_{t-h}^t \dot{\chi}(s)^T (I_N \otimes R) \dot{\chi}(s) ds \leq -\zeta^T(t) W^T F W \zeta(t), \tag{13}$$

where

$$\begin{aligned}
 W &= \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}, W_1 = [0 \quad I \quad -I \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0], \\
 W_2 &= [I \quad -I \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0], \\
 \zeta(t) &= \text{col}\{\chi(t), F(t), \chi(t - h), e^1(t), e^2(t), e^3(t), q(t_kh), q(t_{k-1}h), q(t_{k-2}h)\}.
 \end{aligned}$$

Theorem 3.1. For given positive constants $h, \rho_m, \rho_p, \lambda_s$, and matrix \hat{K}_s , the MAS Eq. (2) subject to deception attack can achieve consensus, if there exists matrices $P > 0, Q > 0, U > 0, R > 0, \Phi > 0$ and symmetrical matrices Z_m, Z_p with appropriate dimensions such that

$$\Pi_{mp} - Z_m < 0, (m, p = 1, 2, \dots, r), \tag{14}$$

$$\rho_p \Pi_{mp} + \rho_m \Pi_{pm} - \rho_p Z_m - \rho_m Z_p + Z_m + Z_p < 0, (m \leq p), \tag{15}$$

$$\begin{bmatrix} I_N \otimes R & 0 \\ 0 & 3(I_N \otimes R) \end{bmatrix} > 0, \tag{16}$$

where

$$\Pi_{mp} = \begin{bmatrix} \Xi_{11} & * & * \\ \Xi_{21} & -(I_N \otimes R)^{-1} & * \\ \Xi_{31} & 0 & -(I_N \otimes P)^{-1} \end{bmatrix},$$

$$\Xi_{11} = \begin{bmatrix} \Psi_1 & * & * \\ \Psi_2 & \Psi_3 & * \\ \Psi_4 & \Psi_5 & \Psi_6 \end{bmatrix},$$

$$\Xi_{21} = [h(I_N \otimes \tilde{A}) \quad -h(H \otimes \tilde{B}\hat{K}_s) \quad 0 \quad \Gamma_1 \quad \Gamma_2],$$

$$\Xi_{31} = [I_N \otimes G \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0],$$

$$\Psi_1 = \begin{bmatrix} \Psi_{11} & * & * \\ \Psi_{21} & \Psi_{22} & * \\ 0 & I_N \otimes R & -I_N \otimes (Q + R) \end{bmatrix},$$

$$\Psi_2 = [col_3\{-2(H \otimes \tilde{B}\hat{K}_s)^T(I_N \otimes P)\} \quad col_3\{\frac{2}{3}(H^T \Lambda H \otimes I_n)\Phi\} \quad 0],$$

$$\Psi_3 = col\{\mathfrak{N}_1 \quad \mathfrak{N}_1 \quad \mathfrak{N}_1\} - diag_3\{\lambda_s \Phi\}, \mathfrak{N}_1 = col_3\{\frac{4}{9}(H^T \Lambda H \otimes I_n)\Phi\},$$

$$\Psi_4 = [col_3\{(C \otimes \tilde{B}\hat{K}_s)^T(I_N \otimes P)\} \quad col_3\{-\frac{1}{3}(C^T \Lambda H \otimes I_n)\Phi\} \quad 0],$$

$$\Psi_5 = col\{\mathfrak{N}_2 \quad \mathfrak{N}_2 \quad \mathfrak{N}_2\}, \mathfrak{N}_2 = col_3\{-\frac{2}{9}(C^T \Lambda H \otimes I_n)\Phi\},$$

$$\Psi_6 = diag_3\{\frac{1}{9}(C^T \Lambda C \otimes I_n)\Phi - I_N \otimes P\}, \Lambda = diag_N\{\delta_i\}, \Phi = diag_n\{\Phi_i\},$$

$$\Psi_{11} = (I_N \otimes \tilde{A})^T(I_N \otimes P) + (I_N \otimes P)(I_N \otimes \tilde{A}) + (I_N \otimes (Q + U - 3R)),$$

$$\Psi_{21} = -(H \otimes \tilde{B}\hat{K}_s)^T(I_N \otimes P) + 3(I_N \otimes R), \Psi_{22} = (H^T \Lambda H \otimes I_n)\Phi - 4(I_N \otimes R),$$

$$\Gamma_1 = [-2h(H \otimes \tilde{B}\hat{K}_1) \quad -2h(H \otimes \tilde{B}\hat{K}_2) \quad -2h(H \otimes \tilde{B}\hat{K}_3)],$$

$$\Gamma_2 = [h(C \otimes \tilde{B}\hat{K}_1) \quad h(C \otimes \tilde{B}\hat{K}_2) \quad h(C \otimes \tilde{B}\hat{K}_3)].$$

Proof. Define the following Lyapunov-Krasovskii functional

$$V(t, \chi(t), \dot{\chi}(t)) = V_1(t, \chi(t)) + V_2(t, \chi(t)) + V_3(t, \chi(t), \dot{\chi}(t)), \tag{17}$$

where

$$V_1(t, \chi(t)) = \chi^T(t)(I_N \otimes P)\chi(t),$$

$$V_2(t, \chi(t)) = \int_{t-h}^t \chi^T(s)(I_N \otimes Q)\chi(s)ds + \int_{t-\tau(t)}^t \chi^T(s)(I_N \otimes U)\chi(s)ds,$$

$$V_3(t, \chi(t), \dot{\chi}(t)) = h \int_{-h}^0 \int_{t+\beta}^t \dot{\chi}^T(s)(I_N \otimes R)\dot{\chi}(s)dsd\beta.$$

Taking the derivative of $V(t, \chi(t), \dot{\chi}(t))$ along the trajectory of the system Eq. (11), one obtains

$$\dot{V}(t, \chi(t), \dot{\chi}(t)) = \dot{V}_1(t, \chi(t), \dot{\chi}(t)) + \dot{V}_2(t, \chi(t)) + \dot{V}_3(t, \chi(t), \dot{\chi}(t)), \tag{18}$$

where

$$\begin{aligned} \dot{V}_1(t, \chi(t), \dot{\chi}(t)) &= \dot{\chi}^T(t)(I_N \otimes P)\chi(t) + \chi^T(t)(I_N \otimes P)\dot{\chi}(t), \\ \dot{V}_2(t, \chi(t)) &= \chi^T(t)(I_N \otimes Q + I_N \otimes U)\chi(t) - \chi^T(t-h)(I_N \otimes Q)\chi(t-h), \\ \dot{V}_3(t, \chi(t), \dot{\chi}(t)) &= h^2 \dot{\chi}^T(t)(I_N \otimes R)\dot{\chi}(t) - h \int_{t-h}^t \dot{\chi}^T(s)(I_N \otimes R)\dot{\chi}(s)ds. \end{aligned}$$

From Lemma 3.1, we have

$$-h \int_{t-h}^t \dot{\chi}^T(s)(I_N \otimes R)\dot{\chi}(s)ds \leq \zeta^T(t) \begin{bmatrix} W_1 \\ W_2 \end{bmatrix}^T \begin{bmatrix} I_N \otimes R & * \\ 0 & 3(I_N \otimes R) \end{bmatrix} \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} \zeta(t), \tag{19}$$

where W_1, W_2 and $\zeta(t)$ are defined in Lemma 3.1.

The event triggering condition in Eq. (5) is equivalent to

$$\sum_{s=1}^3 \sum_{i=1}^N \lambda_s e_i^{sT}(t) \Phi_i e_i^s(t) \leq \sum_{i=1}^N \delta_i \left[\frac{1}{3} \sum_{s=1}^3 \eta_i(t_{k-s+1}^i h) \right]^T \Phi_i \left[\frac{1}{3} \sum_{s=1}^3 \eta_i(t_{k-s+1}^i h) \right]. \tag{20}$$

Thus, we have

$$\begin{aligned} \sum_{s=1}^3 \lambda_s e_i^{sT}(t) \Phi e^s(t) &\leq \left[HF(t) + \frac{2}{3}H \sum_{s=1}^3 e^s(t) - \frac{1}{3}C \sum_{s=1}^3 q(t_{k-s+1}h) \right]^T (\Lambda \otimes I_n) \Phi \\ &\quad \left[HF(t) + \frac{2}{3}H \sum_{s=1}^3 e^s(t) - \frac{1}{3}C \sum_{s=1}^3 q(t_{k-s+1}h) \right]. \end{aligned} \tag{21}$$

Combining Eqs. (17)–(21), it follows that

$$\dot{V}(t, \chi(t), \dot{\chi}(t)) \leq \sum_{m=1}^r \sum_{p=1}^r h_m h_p^{ks} \zeta^T(t) \Omega_{mp} \zeta(t), \tag{22}$$

where $\Omega_{mp} = \Xi_{11} + \Xi_{21}^T(I_N \otimes R)\Xi_{21} + \Xi_{31}^T(I_N \otimes P)\Xi_{31}$.

Taking slack matrix Z_m into account, it yields that

$$\begin{aligned} \sum_{m=1}^r \sum_{p=1}^r h_m h_p^{ks} \zeta^T(t) \Omega_{mp} \zeta(t) &\leq \sum_{m=1}^r \sum_{p=1}^r h_m h_p \zeta^T(t) [\rho_m(\Omega_{mp} - Z_m) + Z_m] \zeta(t) \\ &\quad + \sum_{m=1}^r \sum_{m < p} h_m h_p \zeta^T(t) [\rho_p(\Omega_{mp} - Z_m) + \rho_m(\Omega_{pm} - Z_p) + Z_m + Z_p] \zeta(t). \end{aligned} \tag{23}$$

From $h_p^{ks} - \rho_p h_p \geq 0$ in Eq. (8) for all p and Eqs. (14)-(15), it follows that $\sum_{m=1}^r \sum_{p=1}^r h_m h_p^{ks} \Omega_{mp} < 0$.

Utilizing the Schur complement to Eqs. (14)-(15) and (23), one can conclude that

$$\dot{V}(t, \chi(t), \dot{\chi}(t)) < 0, t \in [\vartheta h, (\vartheta + 1)h]. \tag{24}$$

Therefore, it can be concluded that the MAS Eq. (10) is asymptotically stable, which implies that a gradual consensus is achieved between the states of all followers and the leader. The proof is completed. \square

In the following text, we are in position to co-design controller gains and the parameters of METS for the system Eq. (2) subject to deception attacks.

Theorem 3.2. Under the proposed METS Eq. (5), for some given positive constants $h, \rho_m, \rho_p, \lambda_s$ and μ , the leader-following consensus of MAS Eq. (2) can be achieved, if there exists matrices $\check{Q} > 0, \check{U} > 0, \check{R} > 0, \check{\Phi} > 0$ and symmetrical matrices \check{Z}_m, \check{Z}_p , such that

$$\check{\Pi}_{mp} - \check{Z}_m < 0, (m, p = 1, 2, \dots, r), \tag{25}$$

$$\rho_p \check{\Pi}_{mp} + \rho_m \check{\Pi}_{pm} - \rho_p \check{Z}_m - \rho_m \check{Z}_p + \check{Z}_m + \check{Z}_p < 0, (m \leq p), \tag{26}$$

$$\begin{bmatrix} I_N \otimes \check{R} & 0 \\ 0 & 3(I_N \otimes \check{R}) \end{bmatrix} > 0, \tag{27}$$

where

$$\begin{aligned} \check{\Pi}_{mp} &= \begin{bmatrix} \check{\Xi}_{11} & * & * \\ \check{\Xi}_{21} & -2\mu(I_N \otimes X) + \mu^2(I_N \otimes \check{R}) & * \\ \check{\Xi}_{31} & 0 & -I_N \otimes X \end{bmatrix}, \\ \check{\Xi}_{11} &= \begin{bmatrix} \check{\Psi}_1 & * & * \\ \check{\Psi}_2 & \check{\Psi}_3 & * \\ \check{\Psi}_4 & \check{\Psi}_5 & \check{\Psi}_6 \end{bmatrix}, \\ \check{\Xi}_{21} &= [h(I_N \otimes \check{A}X) \quad -h(H \otimes \check{B}Y_{ps}) \quad 0 \quad \check{\Gamma}_1 \quad \check{\Gamma}_2], \\ \check{\Xi}_{31} &= [I_N \otimes GX \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0], \\ \check{\Psi}_1 &= \begin{bmatrix} \check{\Psi}_{11} & * & * \\ \check{\Psi}_{21} & \check{\Psi}_{22} & * \\ 0 & I_N \otimes \check{R} & -I_N \otimes (\check{Q} + \check{R}) \end{bmatrix}, \\ \check{\Psi}_2 &= [\text{col}_3\{-2(H \otimes \check{B}Y_{ps})^T\} \quad \text{col}_3\{\frac{2}{3}(H^T \Lambda H \otimes I_n)\check{\Phi}\} \quad 0], \\ \check{\Psi}_3 &= \text{col}\{\check{\aleph}_1 \quad \check{\aleph}_1 \quad \check{\aleph}_1\} - \text{diag}_3\{\lambda_s \check{\Phi}\}, \check{\aleph}_1 = \text{col}_3\{\frac{4}{9}(H^T \Lambda H \otimes I_n)\check{\Phi}\}, \\ \check{\Psi}_4 &= [\text{col}_3\{(C \otimes \check{B}Y_{ps})^T\} \quad \text{col}_3\{-\frac{1}{3}(C^T \Lambda H \otimes I_n)\check{\Phi}\} \quad 0], \\ \check{\Psi}_5 &= \text{col}\{\check{\aleph}_2 \quad \check{\aleph}_2 \quad \check{\aleph}_2\}, \check{\aleph}_2 = \text{col}_3\{-\frac{2}{9}(C^T \Lambda H \otimes I_n)\check{\Phi}\}, \\ \check{\Psi}_6 &= \text{diag}_3\{\frac{1}{9}(C^T \Lambda C \otimes I_n)\check{\Phi} - I_N \otimes X\}, \Lambda = \text{diag}_N\{\delta_i\}, \check{\Phi} = \text{diag}_N\{\check{\Phi}_i\}, \\ \check{\Psi}_{11} &= (I_N \otimes \check{A}X)^T + (I_N \otimes \check{A}X) + (I_N \otimes (\check{Q} + \check{U} - 3\check{R})), \\ \check{\Psi}_{21} &= -(H \otimes \check{B}Y_{ns})^T + 3(I_N \otimes \check{R}), \check{\Psi}_{22} = (H^T \Lambda H \otimes I_n)\check{\Phi} - 4(I_N \otimes \check{R}), \\ \check{\Gamma}_1 &= [-2h(H \otimes \check{B}Y_{p1}) \quad -2h(H \otimes \check{B}Y_{p2}) \quad -2h(H \otimes \check{B}Y_{p3})], \\ \check{\Gamma}_2 &= [h(C \otimes \check{B}Y_{p1}) \quad h(C \otimes \check{B}Y_{p2}) \quad h(C \otimes \check{B}Y_{p3})]. \end{aligned}$$

Furthermore, the desired controller gain can be computed as $\hat{K}_s = Y_{ps}X^{-1}$, and $\Phi_i = X^{-1}\check{\Phi}_iX^{-1}$.

Proof. Define $X = P^{-1}$, $\mathcal{M} = \text{diag} \left\{ \underbrace{I_N \otimes X, \dots, I_N \otimes X}_9, I_N \otimes I_n, I_N \otimes I_n \right\}$, and $\check{Q} = XQX$, $\check{U} = XUX$, $\check{R} = XRX$, $\check{\Phi}_i = X\Phi_iX$, $Y_{ps} = \hat{K}_sX$, $s = 1, 2, 3$.

Similar to the method in [42] on dealing with the nonlinear item, it is true that

$$-(I_N \otimes \check{R})^{-1} \leq -2\mu(I_N \otimes X) + \mu^2(I_N \otimes \check{R}). \tag{28}$$

Pre- and post-multiplying Eqs. (14)–(16) with \mathcal{M} and its transpose, one can conclude that Eqs. (25)–(27) are sufficient conditions to guarantee Eqs. (14)–(16) hold. Besides, the leader-following consensus is achieved under deception attacks.

According to the analysis above, the design of leader-following consensus control and METS for the MAS subject to deception attacks is presented in the following algorithm: \square

Algorithm 1 Memory-based event-triggered control algorithm.

Require:

- Initialize the parameters of METS in (20);
- Initialize μ ;
- Let T be the lifespan of the MAS (1).

Ensure:

- while** $0 \leq t \leq T$
 - Step 1: Utilize Matlab LMI toolbox to find feasible solutions of Φ_i in (20) and K_{ps} in (7) according to Theorem 3.2;
 - Step 2: The sensors of MAS sample data $x_i(t)$.
 - Step 3: **if** $\zeta_i(t) < 0$ holds, go to Step 4;
 - otherwise**, go to Step 5.
 - Step 4: Drop the current data; Controller holds the last event-triggered instant (ETI) $x_i(t_k^i)$; Go to Step 3.
 - Step 5: **if** $1 \leq k < 3$, **then**
 - use a buffer to store the released data and update the ZOH.
 - else if** $k \geq 3$, **then**
 - stack the most recent released 3 packets in a buffer;
 - compute $\zeta_i(t)$, $e_i^s(t)$, $\bar{x}_i(t_{k-s+1}^i h)$, $\wp(t)$, update the ZOH.
 - end if**
 - Step 5: Update the state $x_i(t)$ at ETI.
 - Step 6: Compute the consensus control $u_i(t)$ with (6).
 - Step 7: Go to Step 2.
 - end**
-

Remark 3.1. The use of cone compensation linearization can reduce the conservatism of the system, but increase the computational complexity at the same time. To solve this problem, inequality Eq. (28) is used to handle the nonlinear item $-(I_N \otimes R)^{-1}$ in Theorem 3.1.

Notably, when setting $\hat{s} = 1$, the proposed METS reduces to the conventional ETS, in which no historical information is used. In order to compare with the proposed triggering scheme, the following corollary can be deduced.

Corollary 1. Consider the leader-following MASs Eq. (2) and memoryless ETS (i.e $s = 1$). Given positive constants $h, \rho_m, \rho_p, \lambda$ and μ , the consensus of error systems Eq. (9) can be achieved, if there exists matrices $\check{P} > 0, \check{Q} > 0, \check{U} > 0, \check{R} > 0, \check{\Phi} > 0$ and symmetrical matrices \check{Z}_m, \check{Z}_p , such that

$$\check{\Pi}_{mp} - \check{Z}_m < 0, (m, p = 1, 2, \dots, r), \tag{29}$$

$$\rho_p \check{\Pi}_{mp} + \rho_m \check{\Pi}_{pm} - \rho_p \check{Z}_m - \rho_m \check{Z}_p + \check{Z}_m + \check{Z}_p < 0, (m \leq p), \tag{30}$$

$$\begin{bmatrix} I_N \otimes \check{R} & 0 \\ 0 & 3(I_N \otimes \check{R}) \end{bmatrix} > 0, \tag{31}$$

where

$$\check{\Pi}_{mp} = \begin{bmatrix} \check{\Xi}_{11} & * & * \\ \check{\Xi}_{21} & -2\mu(I_N \otimes X) + \mu^2(I_N \otimes \check{R}) & * \\ \check{\Xi}_{31} & 0 & -I_N \otimes X \end{bmatrix},$$

$$\check{\Xi}_{11} = \begin{bmatrix} \check{\Psi}_1 & * & * \\ \check{\Psi}_2 & \check{\Psi}_3 & * \\ \check{\Psi}_4 & \check{\Psi}_5 & \check{\Psi}_6 \end{bmatrix},$$

$$\check{\Xi}_{21} = [h(I_N \otimes \check{A}X) \quad -h(H \otimes \check{B}Y_p) \quad 0 \quad -2h(H \otimes \check{B}Y_p) \quad h(C \otimes \check{B}Y_p)],$$

$$\check{\Xi}_{31} = [I_N \otimes GX \quad 0 \quad 0 \quad 0 \quad 0],$$

$$\check{\Psi}_1 = \begin{bmatrix} \check{\Psi}_{11} & * & * \\ \check{\Psi}_{21} & \check{\Psi}_{22} & * \\ 0 & I_N \otimes \check{R} & -I_N \otimes (\check{Q} + \check{R}) \end{bmatrix},$$

$$\check{\Psi}_2 = [-2(H \otimes \check{B}Y_p)^T \quad 2(H^T \Lambda H \otimes I_n) \check{\Phi} \quad 0], \check{\Psi}_3 = 2(H^T \Lambda H \otimes I_n) \check{\Phi} - \lambda \check{\Phi},$$

$$\check{\Psi}_4 = [(C \otimes \check{B}Y_p)^T \quad -(C^T \Lambda H \otimes I_n) \check{\Phi} \quad 0], \check{\Psi}_5 = -2(C^T \Lambda H \otimes I_n) \check{\Phi},$$

$$\check{\Psi}_6 = (C^T \Lambda C \otimes I_n) \check{\Phi} - I_N \otimes X, \Lambda = \text{diag}_N\{\delta_i\}, \check{\Phi} = \text{diag}_N\{\check{\Phi}_i\},$$

$$\check{\Psi}_{11} = (I_N \otimes \check{A}X)^T + (I_N \otimes \check{A}X) + (I_N \otimes (\check{Q} + \check{U} - 3\check{R})),$$

$$\check{\Psi}_{21} = -(H \otimes \check{B}Y_p)^T + 3(I_N \otimes \check{R}), \check{\Psi}_{22} = (H^T \Lambda H \otimes I_n) \check{\Phi} - 4(I_N \otimes \check{R}).$$

Furthermore, the designed controller gain can be obtained as $K_p = Y_p X^{-1}$, and $\Phi_i = X^{-1} \check{\Phi}_i X^{-1}$.

Proof. Choose the Lyapunov function Eq. (17) and along the proving route of Theorem 3.1, the results can be obtained. For brevity, it's omitted here. \square

4. An illustrative example

A simulation example with two cases is introduced in this section to demonstrate the effectiveness of the proposed consensus control method under METS.

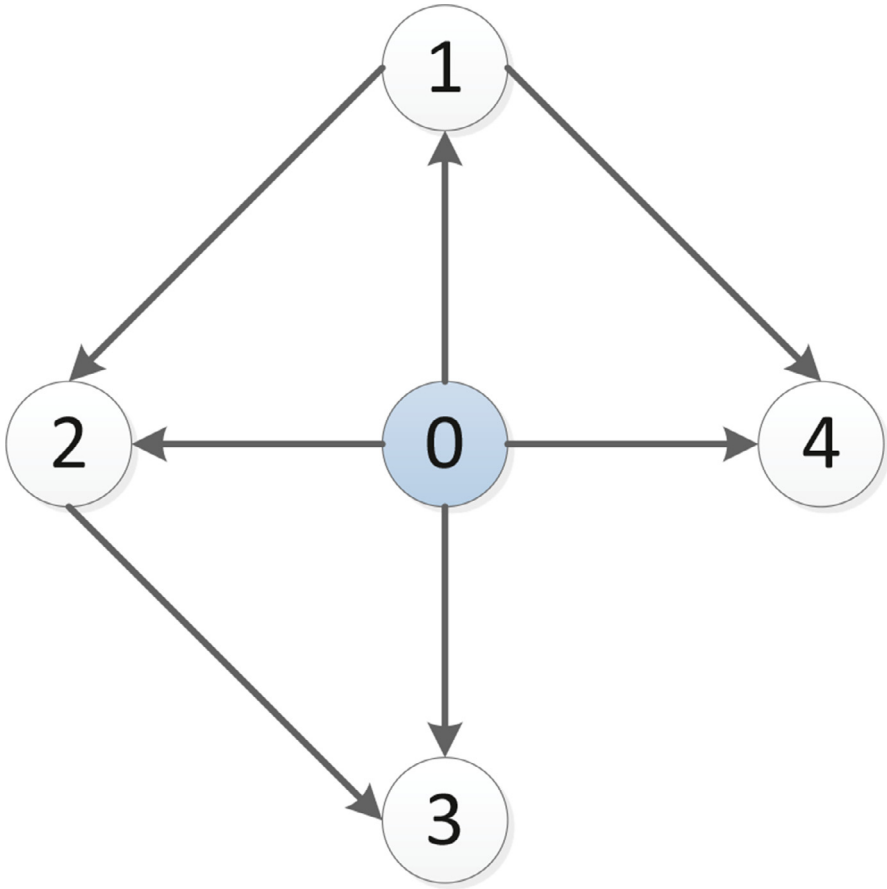


Fig. 2. Topology graph for the MASs.

The MAS with four followers and one leader is considered [43]. The parameters are given as follows:

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, A_2 = \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix}, B_1 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, B_2 = \begin{bmatrix} -1 & 0 \\ 0.1 & -1 \end{bmatrix}.$$

The membership functions $h_1(z(t))$ and $h_2(z(t))$ are assumed as

$$h_1(z(t)) = \frac{1}{1 + e^{x_1(t)-0.5}}, h_2(z(t)) = 1 - h_1(z(t)).$$

The directed topology graph of the communication network is depicted in Fig. 2, in which the leader agent is indicated by 0 while 1–4 stand for follower agents. The weights between agents are equal to one.

The corresponding Laplacian matrix L and leader adjacency matrix D can be obtained as:

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

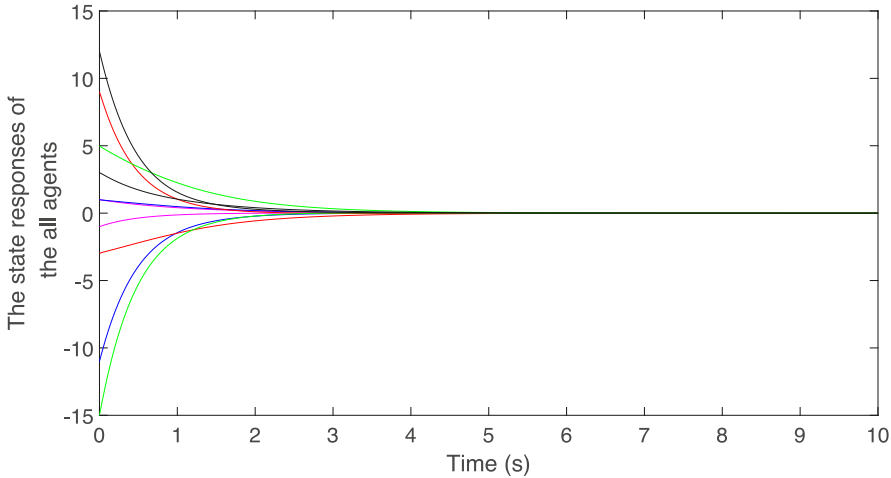


Fig. 3. The trajectories of the states for all followers and the leader with $s = 3$.

Suppose the attack signal function is $q_j(t) = \begin{bmatrix} -\tanh(0.02x_{j1}(t)) \\ -\tanh(0.1x_{j2}(t)) \end{bmatrix}$, and select weights as

$$G_j = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

Case 1: We use 3 historical packets for METS to design the controller. By using Theorem 3.2, we can get the triggering matrices $\Phi_1 = \begin{bmatrix} 0.0564 & -0.0830 \\ -0.0830 & 4.8688 \end{bmatrix}$, $\Phi_2 = \begin{bmatrix} 4.7782 & 0.0522 \\ 0.0522 & 0.4927 \end{bmatrix}$, $\Phi_3 = \begin{bmatrix} 1.1319 & 0.0245 \\ 0.0245 & 4.6388 \end{bmatrix}$, $\Phi_4 = \begin{bmatrix} 0.1891 & 0.0034 \\ 0.0034 & 4.6953 \end{bmatrix}$, and controller gains K_{ps} for $p = 1, 2, s = 1, 2, 3$ as follows:

$$K_{11} = \begin{bmatrix} 0.0018 & 0.0020 \\ 0.0195 & -0.0372 \end{bmatrix}, K_{12} = \begin{bmatrix} -0.0148 & -0.0292 \\ 2.2550 & -0.7495 \end{bmatrix}, K_{13} = \begin{bmatrix} 0.0129 & 0.0734 \\ -0.0008 & 0.0025 \end{bmatrix},$$

$$K_{21} = \begin{bmatrix} 0.0013 & -0.0184 \\ -0.2327 & 0.0615 \end{bmatrix}, K_{22} = \begin{bmatrix} -0.0072 & -0.0156 \\ 0.0003 & -0.0056 \end{bmatrix}, K_{23} = \begin{bmatrix} -0.0135 & -0.0093 \\ 0.0032 & -0.0103 \end{bmatrix}.$$

Under the initial states $x_0(t) = [-1 \ 1]^T$, $x_1(t) = [-11 \ 1]^T$, $x_2(t) = [9 \ -3]^T$, $x_3(t) = [-15 \ 5]^T$, $x_4(t) = [12 \ 3]^T$ and event-triggered parameters $\delta_1 = 0.1, \delta_2 = 0.2, \delta_3 = 0.1, \delta_4 = 0.15, \lambda_1 = 0.5, \lambda_2 = 0.3, \lambda_3 = 0.2$, sampling period $h = 0.1s$, we can get the following responses which are presented in Figs. 3–6.

Figure 3 depicts the state trajectories of all follower agents and the leader, where one can see that the motion of leader and follower agents are identical from 4.7s, that is to say, the leader-following consensus of the fuzzy model-based MAS is achieved. The error responses are drawn in Figs. 4–5, from which one can see that the leader-following consensus has been achieved eventually under the proposed control law. Release intervals for all follower agents under proposed METS are plotted in Fig. 6.

Case 2: We set $s = 1$ and consider the conventional ETS for the leader-following consensus of T–S fuzzy MASs.

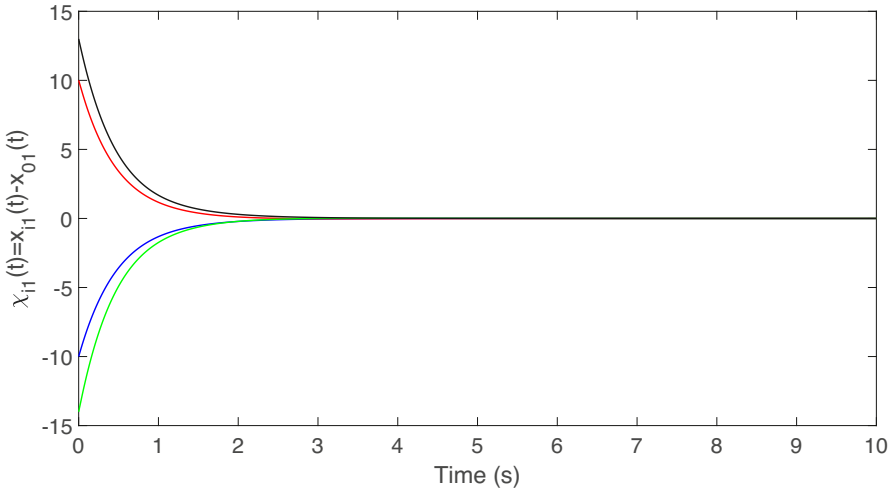


Fig. 4. Consensus error of $\chi_{i1}(t)$ for $i = 1, 2, 3, 4$ under Case 1.

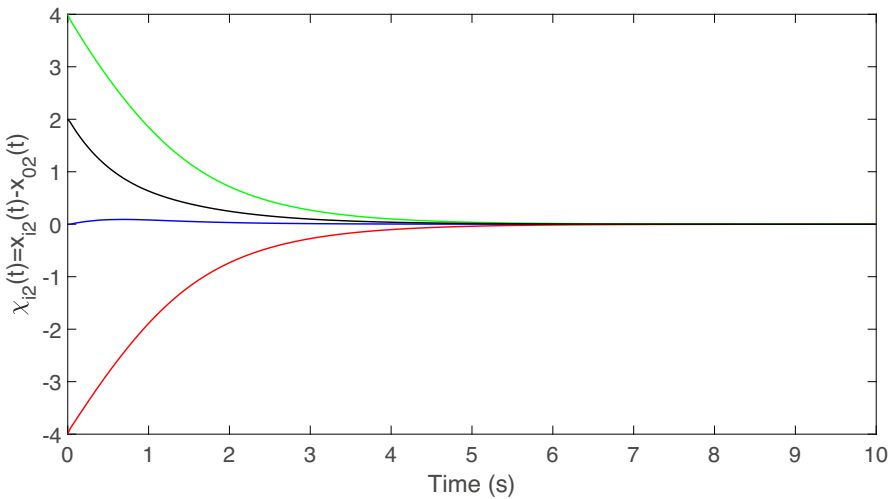


Fig. 5. Consensus error of $\chi_{i2}(t)$ for $i = 1, 2, 3, 4$ under Case 1.

The initial conditions in this case are chosen as the same case 1, and the controller gains K_p , triggering matrices Φ_i are derived by Corollary 1 as follows:

$$\Phi_1 = \begin{bmatrix} 0.3575 & -0.0046 \\ -0.0046 & 0.0144 \end{bmatrix}, \Phi_2 = \begin{bmatrix} -0.0019 & -0.0034 \\ -0.0034 & 0.0049 \end{bmatrix}, \Phi_3 = \begin{bmatrix} 0.0083 & -0.0129 \\ -0.0129 & 0.0126 \end{bmatrix},$$

$$\Phi_4 = \begin{bmatrix} 0.0029 & -0.0454 \\ -0.0454 & -0.0406 \end{bmatrix}, K_1 = \begin{bmatrix} -0.0528 & 0.0193 \\ 0.0024 & -0.0008 \end{bmatrix}, K_2 = \begin{bmatrix} -0.0175 & 0.0138 \\ 0.2973 & -0.2487 \end{bmatrix}.$$

Figure 7 and 8 depict the states of all the followers and the releasing intervals of all the agents.

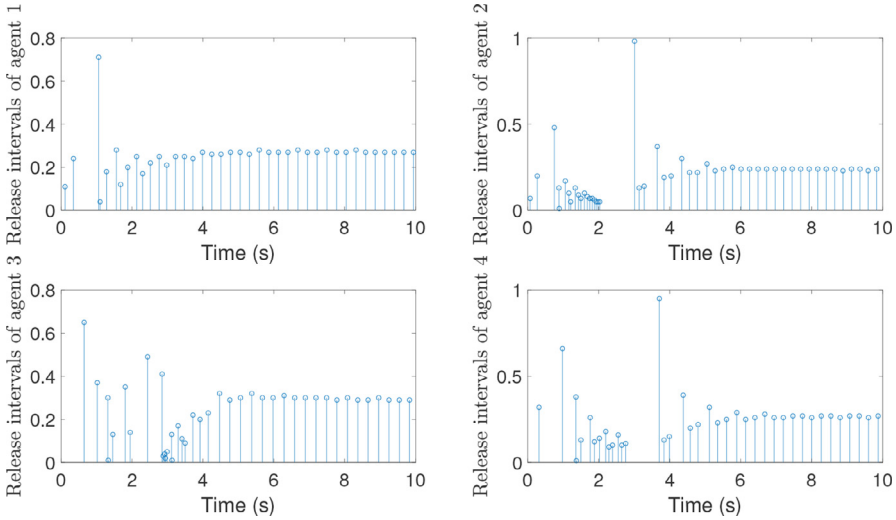


Fig. 6. Release intervals for all followers under Case 1.

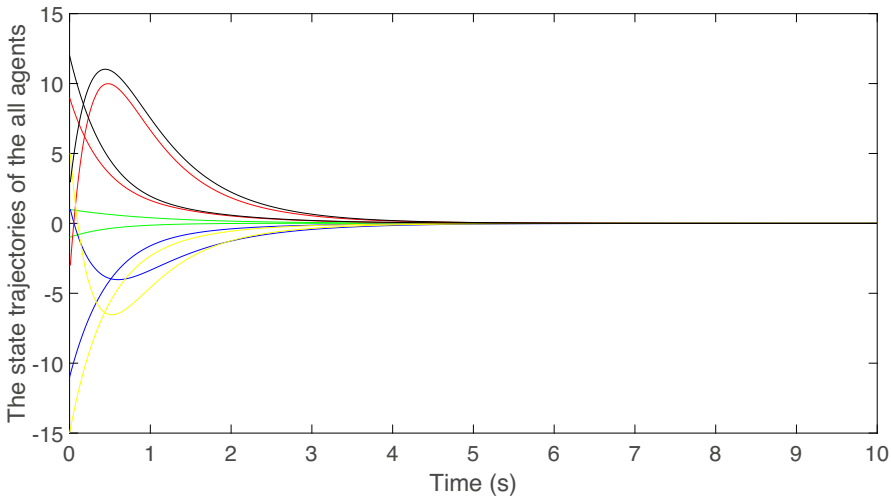


Fig. 7. The trajectories of the states for all followers and the leader under Case 2.

In view of the fact that the system needs to utilize the mutual communication between multi-agents to adjust the following state before stability, we can see that more packets are released under METS at the beginning than the one in case 2 by comparing Figs. 6 and 8. When the consensus of MAS is achieved, the triggering frequency and data releasing rate are much less than the general ETS. What’s more, the historical data packets are applied to adjust the consensus errors among agents so that the increased computation is traded off for the reduced communication burden. The triggering numbers of 4 followers under different ETSs are recorded in Table 1, from which we can clearly see that the proposed METS Eq. (5) has less triggering numbers than ETS in [44]. Therefore, our proposed scheme can effectively reduce the network burden while ensuring the control performance.

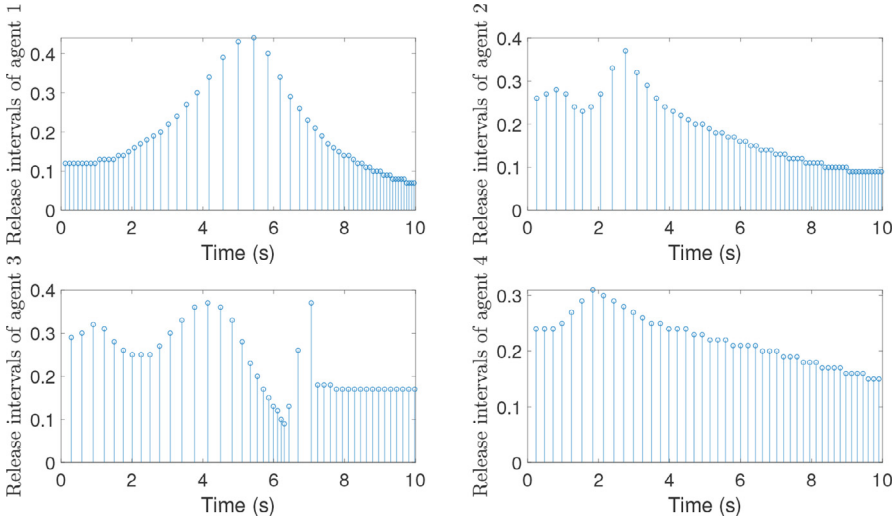


Fig. 8. Release intervals of all the followers under Case 2.

Table 1
Triggering numbers under different schemes.

Triggering schemes	Agent 1	Agent 2	Agent 3	Agent 4
ETS in [44]	61	62	46	47
METS Eq. (5)	40	50	41	40

5. Conclusion

In this paper, we have investigated the T–S fuzzy model-based leader-following consensus for multi-agent systems subject to deception attacks. By applying a novel METS, the data-releasing rate can be greatly reduced, while the efficiency of data releasing is enhanced during the MASs under deception attacks. With the aid of Lyapunov-Krasovskii technique, some sufficient conditions have been proposed to ensure the leader-following consensus of the MASs subject to a kind of deception attacks. The main study orientation of our future work is how to extend the proposed results to cooperative output regulation problem for fuzzy MASs under cyber attacks. Besides, practical experiments will be used to verify the effectiveness of the proposed method in the follow-up work.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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